**FOR FIN 4310 MANAGERIAL FINANCE**

**LECTURE 9**

***FINANCIAL ANALYSIS, CORPORATE VALUATION AND CAPITAL FORMATION***

by

###### OSCAR VARELA, Ph.D.

###### Charles R. and Dorothy S. Carter Chair in Business Administration and Professor of Finance

###### The University of Texas at El Paso

Department of Economics and Finance

El Paso, TX 79968-0543, USA

Ph 915-747-7771, Fax 915-747-6282

Email ovarela3@utep.edu

Emeritus Professor of Finance, University of New Orleans

CFA, Institute of Chartered Financial Analysts

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**LECTURE 9. CAPITAL STRUCTURE, MODIGLIANI‑MILLER ARBITRAGE AND DEBT FINANCING**

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Much has been written about how the firm acquires physical capital via its use of debt and equity financing, its capital structure. The major theoretical work in this area by Modigliani and Miller (1958, 1963) who argue that the capital structure of the firm is irrelevant in determining its value in a no tax work.[[1]](#footnote-1) Arbitrage arguments are used to show that firms otherwise similar in all respects except for their capital structures will provide arbitrage opportunities if one is overvalued and the other is undervalued (in a relative sense). Overvalued firms should be sold (sell high) and undervalued ones bought (buy low), and this profitable arbitrage activity will ultimately lead to an equalization of values, and elimination of arbitrage profits, and the irrelevance of capital structure in determining such values.

In a tax world in which interest payments are tax deductible, Modigliani and Miller show that firms that use debt financing will have higher values than firms similarly situated in all respects except they use no debt financing, because levered firms receive government subsidies via interest tax deduction related to their debt financing. Again, arbitrage arguments are provided that show the exact relationship between the values of unlevered and levered firms when interest payments are tax deductible, such that levered firms should have values that are higher than unlevered firms by the tax rate times the value of their debt (bonds).

The no tax and tax world environments explicitly showing the Modiligani-Miller arbitrage arguments are presented below. We begin with the no tax world environment.

**9.2. Levered Firm Undervalued with No Taxes**

Assume that firms A and B are similar in all respects except for financial leverage. This assumption is partly reflected by the same $20,000 net operating income (NOI) streams for both firms. Firm A has no leverage whereas Firm B has $100,000 of debt at a 5% cost of debt ($20,000 interest cost). Firm As cost of equity (equity capitalization rate) is 8% and Firm Bs (owning to its financial leverage) is higher at 12%. The interest cost, earnings available to stockholders, and the values of stock and debt (assume perpetual cash flows without growth) are shown in Table 9.1 below. The value of stock is the earnings available to stockholders capitalized at the cost of equity, and the value of debt is the interest amount capitalized at the cost of debt. Debt is not amortized and is perpetual (like the British “consol”). Observe that levered firm B is at present undervalued.

**Table 9.1. Levered Firm Undervalued with No Taxes**

One Possible

A B Firm B Equilibrium

Net Operating Income (NOI) $20,000 $20,000 $20,000

Interest ‑ 5,000 5,000

Earnings Available to

Stockholders $20,000 $15,000 $15,000

Equity Capitalization Rate 8% 12% 10%

Value of Stock 250,000 125,000 150,000

Value of Debt (5%) - 100,000 100,000

Total Value $250,000 $225,000 $250,000

Note. The NOI is assumed to be perpetual without growth, and the debt is a “consol”, also perpetual.

### Levered and undervalued firm B is “cheap” in that it is the same in all respects as firm A except that it is levered and undervalued. As a result, an arbitrage opportunity exits such that investors would find it profitable to switch from firm A to firm B. The arbitrage

is executed as follows, assuming that an investor initially owns 10% of firm A.

1. Initial Conditions

An investor owns 10% of Stock A, or $25,000 worth of the stock, or 10% of the $250,000 total value of the stock.

Return on this investment is $2,000, or 10% of $20,000 earnings available to stockholders, or ($25,000 stock ownership / $250,000 stock value) times the $20,000 in earnings available to stockholders.

2. The investor sells their ownership interest in A for $25,000 as A is overvalued relative to B and otherwise it is the same in all other respects.

3. The investor buys an ownership interest in B because B is undervalued relative to A and otherwise it is the same in all other respects. There are various options in how the investor can buy B.

**9.2.1. Arbitrage Option 1**

Invest the entire $25,000 proceeds from selling A in B, and earn a higher return.

The investment of $25,000 in B is invested in the proportion of debt and equity in B.

Therefore, buy the following amounts of B’s bonds and stocks.

Bonds of B = [wd] ($25,000)

= [$100,000/$225,000] ($25,000)

= .44444($25,000)

= $11,111 Notice that this amount equals

11.11% (or $11,111/$100,000)

of B’s debt, and not 10%

Stock of B = [we] ($25,000)

= [$125,000/$225,000] ($25,000)

= .55556($25,000)

= $13,889 Notice that this amount equals

11.11% (or $13,889/$125,000)

of B’s equity, and not 10%

It should be mentioned that the investment in B’s debt (bonds) as shown below neutralizes the debt of B, because the owner is also the lender. Hence, even though B shows debt on its books, it really has none at all (as owners are loaning to themselves). The important thing, however, is that the investor is unambiguously better off with B than with A.

The return from $25,000 investment in B, with $11,111 in bonds and $13,889 in stocks, is:

Return is $2,222.23, or .05 ($11,111) + .12 ($13,889)

or $555.55 + $1,666.68

Notice that for the same $25,000 investment the return is higher with B than with A, such that the investor gains arbitrage profits from switching from overvalued firm A to undervalued firm B.

**9.2.2. Arbitrage Option 2**

The investor can invest less than $25,000 in B, and earn the same return as had been earned in A. In this case, it is necessary to calculate how much less to invest while earning the same return. To do this, first calculate the weighted average return in B (koB), such that:

koB = (wdB) (kdB) + (weB) (keB) =

8.8889% = (.44444) 5% + (.55556) 12%

Therefore, to earn $2,000 return in B (that is, the same dollar return as was earned in A) one must invest $22,499.72 in B, an amount less than the $25,000 invested in A, as shown below.

This amount is obtained by capitalizing the $2,000 return by the weighted average return in B, such that:

$22,499.72 = $2,000 / koB = $2,000 / 8.8889%

An alternative approach as will be shown later is for the investor to simply buy 10% of B’s debt and equity, that is the same proportion of B’s debt and equity as the investor owned in A.

Now, the investor should invest $22,499.72 in B in the proportion of debt and equity in B. Accordingly, out of the $22,499.72, the investor buys the following amounts of B’s bonds and stocks (any small errors are due to rounding off of the weights).

Bonds of B = [wd] ($22,499.72)

= [$100,000/$225,000] ($22,499.72)

= .44444($22,499.72)

= $9,999.78

Notice that the amount $9,999.78 equals 10% (or $9,999.78/$100,000 of B’s debt.

Stock of B = [we] ($22,499.72)

= [$125,000/$225,000] ($22,499.72)

= .55556($22,499.72)

= $12,499.94

Notice that the amount $12,499.94 equals 10% (or $12,499.94/$125,000) of B’s equity, a proportion that is the same as the investor owned as in A.

The return from the $22,499.72 investment in B, with $9,999.78 in bonds and $12,499.94 in stocks, is $2,000 (small errors are due to rounding off of the weights), such that:

Return is $2,000, or .05 ($9,999.78) + .12 ($12,499.94)

or $499.99 + $1,499.99

Notice that the same $2,000 return is obtained with a smaller investment in B than in A.

### 9.2.3. The Arbitrage Result

The investor selling A reduces the value of A and buying B increases the value of B. Ultimately, the two firms must have the same value; otherwise, in efficient and pure markets this investor arbitrage would continue.

One possible equilibrium result for firms A and B, although not the only one, is shown in the last column of Table 9.1. Firm B in this equilibrium example has $20,000 in net operating income and $5,000 in interest, with underlying debt of $100,000 at 5% interest. The earnings available to stockholders is $15,000, and the equity capitalization rate of 10% produces a $150,000 value of stock. The equity capitalization rate of 10% is higher than the interest rate on debt of 5%, as it should be, as the owners have higher risk than the lenders.

Moreover, the equity capitalization rate is such that its higher value exactly offsets the firm’s use of lower cost debt, such as the $250,000 value of levered firm B is the same as that of unlevered firm A. When the value of these two firms is the same arbitrage profits are not possible. The Modigliani-Miller theorem from this arbitrage result is that the value of the firm is independent of its capital structure in a no tax world.

Notice that in this equilibrium situation, the equity capitalization rate for levered firm B (its levered cost of equity), keB is equal to:

keB = keA + (keA - kd)(B/S)

where B is the bond value and S is the stock value for the levered firm.

Generically, in terms of an unlevered (ul) and levered (l) firm, this theorem can be expressed as

ke,l = ke,ul + (ke,ul - kd)(B/S)

where ke,ul is the equity capitalization rate for the unlevered firm (the unlevered cost of equity), kd is the cost of debt, B is the bond value and S is the stock value for the levered firm.

Therefore, using the numbers in our example,

keB = ke,l = 10% = 8% + [(8% - 5%) ($100,000/$150,000)]

Also, in the equilibrium situation, the equity capitalization rate for the unlevered firm (the unlevered cost of equity), ke,ul, is equal to the overall capitalization rate for the levered firm (the levered overall cost of capital), ko,l, such that:

ke,ul = ko,l = wd kd + we ke,l

Therefore, using the numbers in our example, and knowing that in equilibrium this theorem states that keA = koB, then generalizing this theorem for unlevered and levered firms, obtain

ke,ul = ko,l

or

8% = 8% = ($100,000/$250,000)(5%) + ($150,000/$250,000)(10%)

or

8% = 8% = .40 (5%) + .60 (10%)

In equilibrium, a firm’s use of lower cost debt raises its cost of equity on its remaining equity such that it exactly offsets its use of lower cost debt, leaving its overall cost of capital the same as its unlevered cost of equity.

**9.3. Levered Firm Overvalued with No Taxes**

Again, assume that firms A and B are similar in all respects except for financial leverage, that both earn $20,000 in net operating income (NOI) streams, that A has no leverage and B has $100,000 of debt at a 5% cost of debt ($20,000 interest cost). The cost of equity for A (equity capitalization rate) is 8% and for B (owning to its financial leverage) is higher at 10%. But noteworthy, unlike the previous example, the cost of equity for B is not as high as the 12% previously used. The interest cost, earnings available to stockholders, and the values of stock and debt (assume perpetual cash flows without growth) are shown in Table 9.2 below. Observe that levered firm B is presently overvalued.

**Table 9.2. Levered Firm Overvalued with No Taxes**

One Possible

A B Firm B Equilibrium

Net Operating Income (NOI) $20,000 $20,000 $20,000

Interest ‑ 5,000 5,000

Earnings Available to

Stockholders $20,000 $15,000 $15,000

Equity Capitalization Rate 8% 8.5714% 10%

Value of Stock 250,000 175,000 150,000

Value of Debt (5%) - 100,000 100,000

Total Value $250,000 $275,000 $250,000

Note. The NOI is assumed to be perpetual without growth, and the debt is a “consol”, also perpetual.

### Levered and overvalued firm B is “expensive” in that it is the same in all respects as firm A except that it is levered and overvalued. As a result, an arbitrage opportunity exits such that investors would find it profitable to switch from firm B to firm A. The arbitrage

is executed as follows, assuming that an investor initially owns 10% of firm B.

1. Initial Conditions

An investor owns 10% of Stock B, or $17,500 worth of the stock, or 10% of the $175,000 total value of the stock.

Return on this investment is $1,500, or 10% of $15,000 earnings available to stockholders, or ($17,500 stock ownership / $175,000 stock value) times the $15,000 in earnings available to stockholders.

2. The investor sells their ownership interest in B for $17,500 as B is overvalued relative to A and otherwise it is the same in all other respects. The investor also borrows an amount equal to 10% of B’s debt, or $10,000, in order to substitute personal leverage for corporate leverage. Effectively, the investor purchases A’s stock on margin, neutralizing the apparent leverage of B, such that any differences in return between these firms are not due to differences in their leverages. The amount available to invest in A is $27,500, or $17,500 (the stock sale amount) + $10,000 (the amount borrowed). Notice that the $27,500 is also equal to 10% of B’s total value. .As before, the important thing is that the investor as shown below will be unambiguously better off switching to A.

3. The investor buys an ownership interest in A because A is undervalued relative to B and otherwise it is the same in all other respects. There are various options in how the investor can buy A.

**9.3.1. Arbitrage Option 1**

The investor can invest the entire $27,500 in A and earn a higher return. Notice that in this case the investor owns a higher percentage of A, that is, 11% or $27,500/$250,000, than the 10% of B that was formerly owned.

The return from the $27,500 investment in A is $1,700, such that:

Return is $1,700, or $2,200 - $500,

or 8% ($27,500) - 5% ($10,000)

or return on stock in A less interest on loan.

Notice that for the investor’s own $17,500 of equity (netting out the $10,000 margin) the return is higher with A than with B. .

**9.3.2. Arbitrage Option 2**

The investor can invest less than $27,500 in A and earn the same return. In this case, the investor owns the same 10% percent of A with $25,000 as the 10% of B that was formerly owned.

Given that the equity capitalization rate in A of 8%, the investor must earn $2,000 to net $1,500 after the $500 in interest on the $10,000 margin. To earn $2,000 at 8% capitalization, the investor must invest $25,000, out of which $15,000 is equity.

The important point in this case is that the investor earns the same $1,500 net return in A as was earned in B with only $15,000 of equity, and that the investor’s own equity of $15,000 in A is less than the $17,500 in B, while the return is the same.

### 9.3.3. The Arbitrage Result

The investor buying A increases the value of A and selling B decreases the value of B. Ultimately, the two firms must have the same value; otherwise, in efficient and pure markets this investor arbitrage would continue. One possible equilibrium result for firms A and B, although not the only one, is shown in the last column of Table 9.2. Firm B in this equilibrium example has $20,000 in net operating income and $5,000 in interest, with underlying debt of $100,000 at 5% interest. The earnings available to stockholders is $15,000, and the equity capitalization rate of 10% produces a $150,000 value of stock. The equity capitalization rate of 10% is higher than the interest rate on debt of 5%, as it should be, as the owners have higher risk than the lenders. The value of the two firms is the same, regardless of differences in their capital structure. The adjustment of the cost of equity and overall cost of capital when the firm is levered is the same as presented in Section 9.1.1.3.

**9.4. Levered Firm Undervalued with Taxes**

Assume that firms A and B are similar in all respects except for financial leverage, that both earn $20,000 in net operating income (NOI) streams, that A has no leverage and B has $30,000 of debt at a 10% cost of debt ($3,000 interest cost). Also assume that a corporate income tax of 30% exists and that interest on debt is tax deductible.

The cost of equity for A (equity capitalization rate) is 15% and for B (owning to its financial leverage) is higher at 17%. The interest cost, earnings available to stockholders, and the values of stock and debt (assume perpetual cash flows without growth) are shown in Table 9.3 below.

In this table, taxes are calculated as a first round on net operating income, regardless of whether the firm pays interest. In the second round, the tax savings on interest are calculated, such that the actual tax liability of the firm is the net of the tax on net operating income and the tax savings on interest.

Observe that levered firm B (third column of table 9.3) *appears* to be overvalued. But in reality it is not, because of the effect of the tax deduction of interest on the firm’s value. This point will be fully explained below.

**Table 9.3. Levered Firm Undervalued with Taxes**

A B\_\_\_\_\_\_

NOI $20,000 $20,000

Taxes (30%) 6,000 6,000

EAT 14,000 14,000

Interest (10%) ‑ 3,000

Tax Saving on Interest (30%) ‑ 900

Earnings Available to

Stockholders $14,000 $11,900[[2]](#footnote-2)

Equity Capitalization Rate 15% 17%

Value of Stock 93,333 70,000

Value of Debt (10% int) - 30,000[[3]](#footnote-3)

Total Value $ 93,333 $100,000

Firm B appears to be overvalued, but at $100,000 in value is really undervalued, because the value of B relative to A does not reflect completely the $9,000 subsidized value that B receives from the government from its tax deduction of interest when compared to A. This point needs to be carefully understood. Both A and B lose value when their net operating incomes are subject to taxation, because then the earnings after taxes are less than would be the case in a no tax world. But when the government allows firms to deduct interest payments against taxable income, then firms that borrow and pay interest will receive a tax subsidy compared to those that do not. If the value of this tax subsidy is not reflected in the value of the levered firm compared to the unlevered firm, then the firms’ values are not in equilibrium and arbitrage is possible. The arbitrage that is possible – switching from A to B - in the example in table 9.3 above is shown below.

1. Initial Conditions

An investor owns 10% of Stock A, or $9,333 worth of the stock, or 10% of the $93,333 total value of the stock.

The return on this investment is $1,400, or 10% ownership interest times $14,000 earnings available to stockholders, or 15% equity capitalization rate times $9,333 equity ownership.

2. The investor sells their ownership interest in A for $9,333 as A is overvalued relative to B and otherwise it is the same in all other respects.

3. The investor buys an ownership interest in B because B is undervalued relative to A and otherwise it is the same in all other respects. There are various options in how the investor can buy B.

**9.4.1. Arbitrage Option 1**

Invest the entire $9,333 proceeds from selling A in B, and earn a higher return.

The investment of $9,333 in B is invested in the proportion of debt and equity in B, except that the debt of B has to be adjusted for the amount that is not subsidized by its interest tax deduction. Therefore, buy the following amounts of B’s bonds and stocks.

Bonds of B = [($30,000 - $9,000)/($100,000 - $9,000)] ($9,333)

= .230769($9,333)

= $2,153.77

Stock of B = [$70,000/($100,000 - $9,000)] ($9,333)

= .7692($9,333)

= $7,179.23

Notice that the weight of debt in B’s capital structure is based not on $30,000 of debt for B but rather on $21,000 of debt, that is, $30,000 minus $9,000. B’s debt of $30,000 at 10% tax deductible interest, that is, $3,000 in deductible interest, means that its interest payment saves $900 in taxes at a 30% tax break, that is 30% times $3,000. The $900 in tax savings effectively provides B with $900 in subsidized interest, which at a 10% cost of debt provides B with $9,000 in free financing courtesy of the government. Thus, B’s total debt of $30,000 has two components. One of these components is the $9,000 of debt financing paid for by the government and the remaining $21,000 which is B’s own true debt. The total true value of B is therefore not the $100,000 shown in table 9.3 but rather $91,000, as this amount nets out the financing value subsidized by the government.

The return from the $9,333 investment in B, with $2,153.77 in bonds and $7,179.23 in stocks, is:

Return is $1,435.85, or .10($2,153.77) + .17($7,179.23)

or $215.38 + $1,220.47

Notice that for the same $9,333 investment the return is higher with B ($1,435.85) than with A ($1,400), such that the investor gains arbitrage profits from switching from overvalued firm A to undervalued firm B.

**9.4.2. Arbitrage Option 2**

The investor can invest less than $9,333 in B, and earn the same return as had been earned in A. In this case, it is necessary to calculate how much less to invest while earning the same return. To do this, first calculate the weighted average return in B (koB), such that:

koB = (wdB) (kdB) + (weB) (keB)

= [($30,000 - $9,000)/($100,000 - $9,000)] 10%

+ [($70,000)/($100,000 - $9,000)] 17%

= (.230769) 10% + (7692) 17%

= 15.3846%

Notice that the weights of debt (wdB) and equity (weB) in B have been adjusted to neutralize the subsidized value of debt provided by the government for B through the tax deduction of interest. This adjustment is made to consider the true capital structure of B, separate from the free financing provided by the government.

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Therefore, to earn $1,400 return in B (that is, the same dollar return as was earned in A) one must invest $9,100 in B, an amount less than the $9,333 invested in A, as shown below.

This amount is obtained by capitalizing the $1,400 return by the weighted average return in B, such that:

$9,100 = $1,400 / koB = $1,400 / .153846

The investment of $9,100 in B is invested in the proportion of debt and equity in B, except that as before the debt of B has to be adjusted for the amount that is not subsidized by its interest tax deduction. Therefore, buy the following amounts of B’s bonds and stocks.

Bonds of B = [($30,000 - $9,000)/($100,000 - $9,000)] ($9,100)

= .230769($9,100)

= $2,100.00

Stock of B = [$70,000/($100,000 - $9,000)] ($9,100)

= .7692($9,100)

= $7,000.00

The return from the $9,100 investment in B, with $2,100.00 in bonds and $7,000.00 in stocks, is $1,400:

Return is $1,435.85, or .10($2,100.00) + .17($7,000.00)

or $210.00 + $1,190.00 = $1,400.00

Notice that for a $9,100 investment in B, which is less than the $9,333 investment in A, the return is the same, such that the investor gains arbitrage profits from switching from overvalued firm A to undervalued firm B.

**9.5. Levered Firm Overvalued with Taxes**

Assume the same initial conditions as in section 9.1.3 for firms A and B, except that the cost of equity for B (owning to its financial leverage) is 16% instead of 17%. The interest cost, earnings available to stockholders, and the values of stock and debt (assume perpetual cash flows without growth) for firms A and B are shown in Table 9.4 below. Observe that levered firm B (third column of table 9.3) is overvalued, as it is worth over $11,000 more than A’s value even though the value of its tax deduction of interest is worth only $9,000.

**Table 9.4. Levered Firm Overvalued with Taxes**

A B\_\_\_\_\_\_

NOI $20,000 $20,000

Taxes (30%) 6,000 6,000

EAT 14,000 14,000

Interest (10%) ‑ 3,000

Tax Saving on Interest (30%) ‑ 900

Earnings Available to

Stockholders $14,000 $11,900[[4]](#footnote-4)

Equity Capitalization Rate 15% 16%

Value of Stock 93,333 74,375

Value of Debt (10% int) - 30,000[[5]](#footnote-5)

Total Value $ 93,333 $104,375

The arbitrage that is possible – switching from B to A - in the example in table 9.4 above is shown below.

1. Initial Conditions

An investor owns 10% of Stock B, or $7,437.50 worth of the stock, or 10% of the $74,375 total value of the stock.

The return on this investment is $1,190.00, or 10% ownership interest times $11,900 earnings available to stockholders, or 16% equity capitalization rate times $7,437.50 equity ownership.

2. The investor sells their ownership interest in B for $7,437.50.

3. The investor buys an ownership interest in A because A is undervalued. There are various options in how the investor can buy A.

**9.5.1. Arbitrage Option 1**

The investor ownership interest in B is $7,437.50 or 10% ownership in B.

Note that when one owns 10% of B, one is responsible for 10% of B’s debt, but B’s debt is not $30,000, but instead $21,000 after considering the tax subsidy on debt. Thus, the owner of 10% of B’s stock is responsible (in a roundabout way) for 10% of B’s debt.

Now, sell ownership interest in B, and receive $7,437.50. Also, borrow an amount equal to 10% of B’s debt, or $2,100 at 10% interest (using homemade leverage).

Thus, from the investor’s point of view, the leverage in A is the same as that in B, and differences in return cannot therefore be related to differences in leverage. The total amount available to invest in A is $9,537.50.

Invest this entire amount of $9,537.50 in A. Notice that in this case the investor owns a higher percentage of A, that is, 10.22% or $9,537.50 / $93,333, than the 10% of B that was formerly owned.

The return from the $9,537.50 investment in A is $1,220.63, such that:

Return is $1,220.63, or $1,430.63 - $210,

or 15% ($9,537.50) - 10% ($2,100)

or return on stock in A less interest on loan.

Notice that for the investor’s $7,437.50 of equity (netting out the $2,100 margin) from the investment in A produces a $1,220.63 return. This is a higher than the $1,190.00 return that was earned in B.

**9.5.2. Arbitrage Option 2**

Invest an amount equal to 10% of A, or $9,333.30, which is less than the $9,537.50 that is available. Given that the equity capitalization rate in A of 15%, the investor must earn $1,400 from the investment in A in order to net $1,190, the amount earned in B, after the $210 in interest on the $2,100 margin is paid. To earn $1,400 at 15% capitalization, the investor must invest $9,333.33, financed with $7,233.33 of the investor’s equity and $2,100 in debt. Thus, the investor can, instead of investing $7,437.50 of equity in B, invest $7,233.33 in A and earn the same $1,190 net return.

### 9.5.3. The Arbitrage Result

The investor buying A increases the value of A and selling B decreases the value of B. Ultimately, the two firms must have the same value net of the amount of financing subsidized by the government; otherwise, in efficient and pure markets this investor arbitrage would continue. One possible equilibrium result for firms A and B, although not the only one, is shown in the Table 9.5. Firm B in this equilibrium example has $20,000 in net operating income and $3,000 in interest, with underlying debt of $30,000 at 10% interest. The earnings available to stockholders is $11,900, and the equity capitalization rate of 16.4517% produces a $72,333 value of stock. The equity capitalization rate of 16.4517% is in equilibrium, as the value of firm B is $102,333 and the value of A is 93,333. The $9,000 higher value of firm B is equal to the value of the financing provided by the government through the tax deduction of interest. Effectively, this is free financing provided by the government, such that in equilibrium levered firms should be worth more than unlevered firms by the value of this free financing, as levered firms take advantage of the tax deduction of interest.

## **Table 9.5. Unlevered and Levered Firms in Equilibrium with Taxes**

A (Firm B equilibrium)

NOI $20,000 $20,000

Taxes (30%) 6,000 6,000

EAT 14,000 14,000

Interest (10%) ‑ 3,000

Tax Saving on Interest (30%) ‑ 900

Earnings Available to

Stockholders $14,000 $11,900[[6]](#footnote-6)

Equity Capitalization Rate 15% 16.4517%

Value of Stock 93,333 72,333

Value of Debt (10% int) - 30,000[[7]](#footnote-7)

Total Value $ 93,333 $102,333

Modigliani and Miller show that in the tax case in equilibrium the traditional equity capitalization rate for the levered firm (the levered cost of equity), ke,l, is equal to:

ke,l = ke,ul + (ke,ul - kd)(B/S)(1 - t)

where ke,ul is the equity capitalization rate for the unlevered firm (the unlevered cost of equity), kd is the cost of debt, B is the bond value and S is the stock value for the levered firm, and t is the tax rate.

Therefore, for the example in Table 9.5,

ke,B = ke,A + (ke,A - kd) (B/S) (1 - t)

16.4517% = 15% + [(15% - 10%) ($30,000/$72,333)(1-.30)]

Also, the traditional weighted average cost of capital for the levered firm B is

WACCB = ke,ul {1 - [(B/S) (taxB)]}.

such that

13.68% = 15% {1 - [(30,000/102,333) (.30)]}

or using an even more traditional approach,

WACCB = (DebtB / ValueB) (kd,B) (1-taxB) + (EquityB/ValueB)(ke,B)

or

WACCB = ($30,000/$102,333)(10%)(1-.30) +

($72,333/$102,333)(16.4517%)

= 13.68%

Now, the actual weighted average cost of capital for the levered firm B requires eliminating from the calculation above the $9,000 of free financing provided by the government. This adjustment is made below, such that

Actual WACCB = [($30,000 - $9,000)/($102,333 - $9,000)] 10%

+ [($72,333)/($102,333 - $9,000)] 16.4517%

= 15%

**9.6. The Miller Model – Capital Structure with Personal Taxes**

The Miller Model generalizes the Modigliani-Miller capital structure theory by introducing personal taxes on an investor’s interest and dividend receipts from investments in a levered firm. This model is illustrated by example at the beginning of this section with its derivation and proof shown at the end. At first a set of initial conditions for a levered fir with corporate but not personal taxes is presented. The results are in accord with what has already been shown in this chapter, namely that a levered firm with corporate taxes in which interest is tax deductible gains value relative to an equivalent unlevered firm equal to the value of the tax subsidy on interest. The investor is initially not subject to any taxes, but subsequently this condition is changed such that the investor is liable for taxes on interest and dividends. The effect of this change on the value to the firm from having debt with interest that is tax deductible is then examined. Subsequently the Miller model is formally derived.

**9.6.1. Initial Conditions with Corporate but not Personal Taxes**

Imagine a firm with (perpetual) debt (VD) of $1,000,000. The interest rate of this debt is 9%, leading to a $90,000 interest charge. Assume that the firm is in the 30% corporate tax rate bracket (tc) and that the interest charge is deductible. The tax savings on the interest charge then is $27,000, that is, 30% times $90,000, and the net interest charge is $63,000, that is $90,000 less $27,000. The value of the tax savings is $300,000, equal to $27,000 / .09. This result is consistent with the Modigliani-Miller tax case, wherein the value to the firm from using debt is equal to the tax rate times the value of its debt, or (tc) (VD), or .30 times $1,000,000.

**9.6.2. Adding Personal Taxes - Illustrating the Miller Model**

Now, following up on the initial conditions described above, introduce a personal tax on interest receipts (tpd) equal to 10%. Of course, the firm still gets to shield $27,000 in its interest expense. However, the individual investors purchasing the firm’s bonds who before paid no tax on interest payments they received must now pay 10% tax on the $90,000 of interest received, or $9,000 in taxes.

Thus, the net interest charge on the debt is the $63,000 the firm pays plus the $9,000 the individual investors pay, or $72,000. The individual investors’ $9,000 tax payment offsets by this amount the firm’s original $27,000 tax shield. Considering this adjustment, the tax shield is now equal to $18,000, that is, the original shield of $27,000 offset by the $9,000 individual tax. With this personal tax on interest, the value of the tax shield which originally equaled $300,000 (or $27,000 / .09) now equals $200,000 (or $18,000 / .09). The personal tax reduces the value of the tax shield.

This example illustrates how personal taxes on interest can alter the initially calculated value of the tax shield in Modigliani and Miller’s original work on capital structure with taxes. The Miller model takes considerations of these adjustments when personal taxes on interest and dividends and realized capital gains exist.

Modifying the Modigliani-Miller model tax case with the Miller model, obtain the value of the firm (VF) as follows

VF = VU + { 1 – [ (1-tc) (1-tps) / (1+tpb) ] } B,

where the first term on the right-hand-side is the value of the unlevered firm, the second term on the right-hand-side is the value of the tax shield, and B is the market value of debt after the individual pays tax on debt interest, such that

B = (kd) (VD) (1-tpb) / (kd)

where in these formulas kd is the cost of debt, VU is the value of the unlevered firm, VD is the firm’s debt, tc is the corporate tax rate, tpb is the personal tax rate on bond interest, and tps is the personal tax rate on stock dividend payments and/or realized capital gains.

In our illustration, following the formula shown above for B and assuming that tpb is 10%, obtain

B = (.09) (1,000,000) (1 - .10) / (.09) = $81,000 / .09 = 900,000

Now, if in addition to tpb being 10%, and assuming that tc is 30% and tps is 0%, then the value of the tax shield following our example, given that B is $900,000, is,

Value of the Tax Shield =

{ 1 – [ (1-tc) (1-tps) / (1-tpb) ] } B,

or

{ 1 – [ (1-.30) (1-.00) / (1-.10) ] } $900,000

= (0.2222222) ($900,000) = $200,000.

This result confirms our previous calculation.

The Miller model can also be used to calculate the value of the tax shield when a personal tax also exists on returns to equity, including dividends and realized capital gains. This more generalized result is illustrated above.

If in addition to tpb being 10% and tc being 30%, assume that tps is 6%. Then the value of the tax shield, given that B is $900,000, increases, such that

Value of the Tax Shield =

{ 1 – [ (1-tc) (1-tps) / (1-tpb) ] } B,

orr

{ 1 – [ (1-.30) (1-.06) / (1-.10) ] } $900,000

= (0.268889) ($900,000) = $242,000.

Now, why is it that when a personal tax on dividends or realized capital gains equal to 6% is added to the analysis does the value of the tax shield rise from $200,000 to $242,000? The main reason for this result is that the higher personal tax on stock returns neutralizes the negative effects of the personal tax on interest to the value of leverage to the firm.

This point can be further emphasized by noticing that if tps = tpb = 10%, such that tps increases from 6% to 10%, then the value of the tax shield (using the Miller formula above) would be even higher at $270,000. In this case, there is a change from the initial $200,000 value of the tax shield to the firm when the personal tax on interest was 10% to $270,000 when the personal taxes on interest and stock returns are 10%. This result is due to the increasing neutrality on the value of the tax shield to the firm when affected by the personal tax on interest (tpb) as the personal tax on dividends and realized capital gains (tps) rises. More specifically, the tax savings to the firm when tps = tpb = 10% is no longer its original $27,000 before personal taxes were considered, but rather $24,300, that is $27,000 less the $2,700 liability given the 10% personal tax on interest. The new tax savings of $24,300 when capitalized at 9% has a value of $270,000 when personal taxes on stock and bond returns are the same. Thus, the value of the tax shield to the firm increases in a world with personal taxes when the personal tax on stock returns is higher, as the tax on stock returns neutralizes the negative effect on the value of the tax shield when a personal tax on interest exists.

Second, notice that even when these personal tax rates on interest and dividends and realized capital gains are the same, the fact that the individual must pay tax on interest means that the value of leverage owing to the tax shield can never be equal to the value of leverage without personal taxes. Rather, all else the same, the value of leverage to the firm is higher the higher the tax rate on dividends and realized capital gains.

In the Miller Model, it turns out that with corporate and personal taxes, the value of the firm VF is

VF = VU + { (kd) (VD) [ (1- tpb) - (1- tc) (1- tps) ] } / (kd)

where the value of the unlevered firm is the first term on the right hand side and the value of the levered firm’s tax shield is the second term on the right hand side.

This expression can be simplified using the following steps, where

VF = VU + [(kd) (VD) (1- tpb)] / (kd) - [(kd) (VD) (1- tc) (1- tps) ] / (kd)

and

VF = VU + B - [(kd) (VD) (1- tc) (1- tps) (1- tpb)] / (kd) (1- tpb)

when B = (kd) (VD) (1-tpb) / (kd), and

VF = VU + B - [(kd) (VD) (1- tpb) (1- tc) (1- tps) ] / (kd) (1- tpb)

VF = VU + B - [(kd) (VD) (1- tpb) / (kd)] [(1- tc) (1- tps) ] / (1- tpb)]

VF = VU + B - [B (1- tc) (1- tps) ] / (1- tpb),

such that

VF = VU + { 1 - [ (1- tc) (1- tps) ] / (1- tpb)} B

Now, if there are no personal taxes, such that tps = tpb = 0, then

VF = VU + { 1 - [ (1- tc) ] } B = tc B

as shown by Modigliani and Miller.

**9.7. Derivation of the Miller Model**

The after tax cash flow stream for an all equity firm is:

NOI (1- tc) (1- tps)

The value of this firm, VU, given the all equity capitalization rate ρ is:

VU = NOI (1- tc) (1- tps) / ρ

Now, with bonds and shares outstanding, we have

Payment to shareholders = [ NOI - (kd) (VD) ] (1- tc) (1- tps)

Payment to bondholders = [ (kd) (VD) ] (1- tpb)

Therefore, the total cash payments to all suppliers of capital are:

Payment to shareholders + Payment to bondholders

= [ NOI - (kd) (VD) ] (1- tc) (1- tps) + (kd) (VD) ] (1- tpb)

= [ NOI ] (1- tc) (1- tps) + { [(kd) (VD) ] (1- tpb) - (kd) (VD) ] (1- tc) (1- tps) }

Then, the value of the firm with leverage, VL, is the capitalized value of these payments, such that

= [ NOI ] (1- tc) (1- tps) / ρ

+ [ (kd) (VD) ] [(1- tpb) / kd ] - [ (kd) (VD) ] (1- tc) (1- tps) / kd ]

= VU + [ (kd) (VD) ] [(1- tpb) / kd ] - [ (kd) (VD) ] (1- tc) (1- tps) / kd ]

or

= VU + [ (kd) (VD) ] [ (1- tpb) - (1- tc) (1- tps) ] / kd ]

Then, defining B as (kd) (VD) (1-tpb) / (kd), obtain the Miller model, such that

= VU + { 1 - [ (1- tc) (1- tps) / (1 - tpb) ] } B

1. Modigliani, Franco and Merton Miller (1958). “The cost of capital, corporation finance and the theory of investment”. American Economic Review 48, 261‑297.

   Modigliani, Franco and Merton Miller (1963). “Corporate income taxes and the cost of capital: A correction”. American Economic Review 53, 433–443. [↑](#footnote-ref-1)
2. This amount equals:

   $20,000 NOI – $6,000 Taxes – $3,000 Interest + $900 Interest Tax Saving; or

   ($20,000 NOI – $3,000 Interest)(1 - .30 Tax Rate) [↑](#footnote-ref-2)
3. This $30,000 of debt may be viewed as composed of $21,000 in B’s debt and $9,000 in added value to B through subsidized government debt. [↑](#footnote-ref-3)
4. This amount equals:

   $20,000 NOI – $6,000 Taxes – $3,000 Interest + $900 Interest Tax Saving; or

   ($20,000 NOI – $3,000 Interest)(1 - .30 Tax Rate) [↑](#footnote-ref-4)
5. This $30,000 of debt may be viewed as composed of $21,000 in B’s debt and $9,000 in added value to B through subsidized government debt. [↑](#footnote-ref-5)
6. This amount equals:

   $20,000 NOI – $6,000 Taxes – $3,000 Interest + $900 Interest Tax Saving; or

   ($20,000 NOI – $3,000 Interest)(1 - .30 Tax Rate) [↑](#footnote-ref-6)
7. This $30,000 of debt may be viewed as composed of $21,000 in B’s debt and $9,000 in added value to B through subsidized government debt. [↑](#footnote-ref-7)